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A local hidden variables model for the measured EPR-type flavour entanglement in $Y(4S) \rightarrow B^0\overline{B^0}$ decays

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Abstract

A local hidden variables model is exhibited which gives predictions in agreement with the quantum ones for the recent experiment by Go et al., quant-ph/0702267 (2007)

The aim of this note is to show that the results of the recent experiment measuring EPR-type flavour entanglement in $Y(4S) \rightarrow B^0\overline{B^0}$ decays[1] are compatible with local realism. It is known that a Bell inequality test cannot be performed[2] but this does not prove that the experiment is compatible with local realism. I shall prove the compatibility by exhibiting a local hidden variables (LHV) model which reproduces the quantum prediction (and agrees with the obtained results within experimental errors). The time-dependent rate for decay into two flavour-specific states are[1]

$$R_i(\triangle t) = \frac{1}{4\tau}\exp(-\triangle t/\tau)\left(1 + (-1)^i \cos(\triangle m \triangle t)\right), \ \triangle t = |t_2 - t_1|, \quad (1)$$

where $\triangle m$ is the mass difference between the two $B^0 - \overline{B^0}$ mass eigenstates, $i = 1$ corresponds to the decays B^0B^0 or $\overline{B^0}\overline{B^0}$ and $i = 2$ to the decays $B^0\overline{B^0}$ or $\overline{B^0}B^0$. Actually R_i are the probability densities of decay at time t_2 of the second particle (say the one going to the right) conditional to the decay of the first (say going to the left) at time t_1, both t_1 and t_2 being proper times of the corresponding particles. For our purposes it is more convenient to consider the joint probability densities, $r_{kl}(t_1, t_2)$ for decay of the first particle at time t_1 and second at time t_2, where $k = 1$ ($l = 1$) means that the first (second) particle decays as B^0 and $k = 2$ ($l = 2$) means that the first (second) particle decays as $\overline{B^0}$.

According to Bell´s definition of LHV model,[3] appropriate for our case, we should attach hidden variables λ_1 and λ_2 to the first and second particles, respectively, and define probability densities ρ, P_k, Q_l such that

$$r_{kl}(t_1, t_2) = \int \rho(\lambda_1, \lambda_2) P_k(\lambda_1, t_1) Q_l(\lambda_2, t_2) d\lambda_1 d\lambda_2. \quad (2)$$

The function ρ, giving the initial distribution of the hidden variables in an ensemble of $Y(4S)$ decays, should be positive and normalized, that is

$$\rho(\lambda_1, \lambda_2) \geq 0, \int \rho(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 = 1. \quad (3)$$

The functions $P_k(\lambda_1, t_1)$ represent the probability density that a particle with label λ_1 decays at time t_1 as a B^0 $\left(\overline{B^0}\right)$ if $k = 1(2)$ and similar for Q_l. Thus these functions should be positive and, as all B^0 or $\overline{B^0}$ particles decay sooner or later, they should be normalized for any $\{\lambda_1, \lambda_2\}$, that is

$$P_k, Q_l \geq 0, \int_0^\infty dt_1 \sum_{k=1}^{2} P_k(\lambda_1, t_1) = 1, \int_0^\infty dt_2 \sum_{l=1}^{2} Q_l(\lambda_2, t_2) = 1. \quad (4)$$

Any choice of functions $\{\rho, P_k, Q_l\}$ fulfilling eqs.(2) to (4) provides a LHV model predicting the joint probabiliy densities of decay $r_{kl}\left(t_1, t_2\right)$.

I propose the following. For the initial distribution of hidden variables

$$\rho\left(\lambda_1, \lambda_2\right) = \frac{1}{4\tau N\left(\lambda_2\right)}\delta\left(\lambda_1 - \lambda_2\right),\ \lambda_1, \lambda_2 \in [0, 2\pi], \tag{5}$$

where $\delta\left(\right)$ is Dirac´s delta, and the functions $N\left(\lambda_2\right)$ will be defined below, after eqs.(8), where the normalization of $\rho\left(\lambda_1, \lambda_2\right)$ will be proved. For the probabilities of decay

$$P_k\left(\lambda_1, t_1\right) = \frac{1}{\tau}\exp\left(-\frac{t_1}{\tau}\right)\Theta\left(t_1\right)\sum_{n=0}^{\infty}\Theta\left(\frac{\pi}{2} - \left|\lambda_1 + (2n-k)\pi - \triangle m t_1\right|\right), \tag{6}$$

$$Q_l\left(\lambda_2, t_2\right) = N\left(\lambda_2\right)\exp\left(-\frac{t_2}{\tau}\right)\Theta\left(t_2\right)\left[\cos\left(\lambda_2 - (l+1)\pi - \triangle m t_2\right)\right]_+, \tag{7}$$

where $\Theta\left(t\right) = 1$ (0) if $t > 0$ $(t < 0)$ and $[x]_+$ means putting 0 if $x < 0$. Thus all four functions are decaying exponentials modulated by periodic funcions which oscillate with period $2\pi/\triangle m$. Physically this means that each particle "lives" as a B^0 during a time interval of duration $\pi/\triangle m$, then becomes a $\overline{B^0}$ during another time interval $\pi/\triangle m$, and so on, until it decays. The particles are always anticorrelated in the sense that, at equal proper times, one of them is B^0 and the other one is $\overline{B^0}$.

From eqs.(6) and (7) it is easy to see that the total probability densities (i. e. independently of flavour) for the decay of particles 1 and 2, are respectively

$$\begin{aligned}
\sum_{k=1}^{2} P_k\left(\lambda_1, t_1\right) &= \frac{1}{\tau}\exp\left(-\frac{t_1}{\tau}\right), \\
\sum_{l=1}^{2} Q_l\left(\lambda_2, t_2\right) &= N\left(\lambda_2\right)\exp\left(-\frac{t_2}{\tau}\right)\left|\cos\left(\lambda_2 - \triangle m t_2\right)\right|,
\end{aligned} \tag{8}$$

where $t_1, t_2 \geq 0$. We see that the decay of the first particle is given by a standard exponential, but the decay law of the second particle is more involved. The functions $N\left(\lambda_2\right)$ are chosen so that the normalization eq.(4) holds true. It is not necessary to calculate explicitly the functions $N\left(\lambda_2\right)$,

which are rather involved, but I derive an important property, namely

$$\begin{aligned}\int_0^{2\pi}\frac{1}{N(\lambda)}d\lambda &= \int_0^{2\pi}d\lambda\int_0^{\infty}\exp\left(-\frac{t}{\tau}\right)\left|\cos\left(\lambda-\triangle mt\right)\right|dt \\ &= \int_0^{\infty}\exp\left(-\frac{t}{\tau}\right)dt\int_0^{2\pi}\left|\cos\left(\lambda-\triangle mt\right)\right|d\lambda = 4\tau. \quad (9)\end{aligned}$$

This relation proves that the distribution $\rho(\lambda_1,\lambda_2)$, eq.(5), is indeed normalized.

In order to get $r_{kl}(t_1,t_2)$ we should insert eqs.(6) and (7) in eq.(2) and perform integrals which are straightforward. Introducing the new variable

$$x=\lambda_1+2n\pi-k\pi-\triangle mt_1, \tag{10}$$

and performing the integral in λ_2, using Dirac´s delta, we get

$$\begin{aligned} r_{kl}(t_1,t_2) &= \frac{1}{4\tau^2}\exp\left(-\frac{t_1+t_2}{\tau}\right)I_{kl}, \\ I_{kl} &= \int_{-\pi/2}^{\pi/2}dx\left[\cos\left(x+(k-l-1)\pi+s\right)\right]_+,\ s\equiv\triangle m(t_1-t_2), \quad (11)\end{aligned}$$

where we have taken into account that only one term of the sum in n may contribute, depending on the values of t_1 and t_2, and we have removed the irrelevant term $2n\pi$ in the argument of the cosinus function. It is easy to see that the functions I_{kl} are periodic in the variable s with period 2π. Thus it is enough to consider the interval $s\in[0,2\pi]$. Thus in the particular cases $k=l=1$ or $k=l=2$ the integral (11) becomes, for $s\in[0,\pi]$

$$I_{11}(t_1,t_2)=I_{22}(t_1,t_2)=\int_{\pi/2-s}^{3\pi/2-s}\left[\cos x\right]_+dx=\int_{\pi/2-s}^{\pi/2}\cos xdx=1-\cos s, \tag{12}$$

and for $s\in[\pi,2\pi]$

$$I_{11}(t_1,t_2)=I_{22}(t_1,t_2)=\int_{\pi/2-s}^{3\pi/2-s}\left[\cos x\right]_+dx=\int_{-\pi/2}^{3\pi/2-s}\cos xdx=1-\cos s. \tag{13}$$

Similarly we get, for any $s=\triangle m(t_1-t_2)$,

$$I_{12}(t_1,t_2)=I_{21}(t_1,t_2)=1+\cos s, \tag{14}$$

Finally we obtain

$$r_{kl}\left(t_1,t_2\right)=\frac{1}{4\tau^2}\exp\left(-\frac{t_1+t_2}{\tau}\right)\left[1-(-1)^{l-k}\cos\left(\triangle m\,\triangle t\right)\right],\triangle t=\left|t_1-t_2\right|. \tag{15}$$

Hence we may get eq.(1) via the equality which defines the conditional probability reported in the commented paper[1] in terms of the joint probability, namely

$$R_i=\tau\exp(2t_i/\tau)r_{kl},\ j=|k-l|+1,$$

where $i=1(2)$ if particle 1(2) is the one decaying first. This proves that our LHV model´s prediction agrees with the quantum one for the said experiment.

The model may be interpreted physically saying the either particle produced in the decay of the Y(4S) oscillates between the two flavour states in such a way that the flavours or the two particles in a pair are opposite at equal proper times. The model looks somewhat contrived due to the lack of symmetry, in the sense that the functions P_k are quite different from the functions Q_l. A more symmetrical model may be obtained assuming that the assignement of the functions P_k and Q_l to the particles in a pair is at random. In any case our purpose was only to show the compatibility of the experiment with local realism, and not to make a physically plausible model.

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References

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